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A Report

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ANALYTICAL STUDY OF TWIN-JET SHIELDING
SEMI-ANNUAL PROGRESS REPORT

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DEVELOPMENT OF AN ANALYTICAL MODEL
OF TWIN JET SHIELDING

1. INTRODUCTION

One of the drawbacks of the growing dependence on air travel is the increased impact of aircraft noise. Assessment and reduction of this impact requires identification of aircraft generated noise levels. To this end, the Noise Technology Branch of NASA/Aircraft Noise Reduction Division is developing and refining an aircraft noise prediction computer program. Noise estimination includes consideration not only of noise sources on the aircraft, but also of the propagation path between source and receiver. One of the factors affecting the noise transmission path is shielding of one jet by another. The shielding jet, because of the high temperature and flow speed with respect to the immediate surroundings, acts as a partial barrier between the shielded jet and the receiver. The resultant alteration of the propagation path not only affects the overall aircraft noise level, but also indicates the possibility of jet engine installation as a means of aircraft noise control.

It is the purpose of this paper to discuss the development of the analytical model to estimate the shielding of one jet by an adjacent jet in a twin jet configuration.

The problem of reflection and transmission of sound by a moving medium has been addressed assuming a plane wave incident on a plane interface (1, 2,3,4). Ray tracing techniques have been applied to two-dimensional jets (8) and cold jets (7).

In the current study, the three - dimensional case is considered. The noise source is a discrete frequency point source at rest with respect to the jet axis. The shielding jet is assumed to be a cylinder of heated flow in which the temperature and flow velocity profiles are constant across the jet.

The three-dimensional character of the noise source is required in order to investigate the effect on shielding of the orientation of the emitting jet with respect to the shielding jet. The finite cross-section of the shielding jet permits investigation of diffraction around the jet. The problem is formulated in three dimensions in order to investigate forward and backward scattering phenomena (7), as well as the influence of jet flow speed.

II FORMULATION AND SOLUTION OF THE MODEL

The mechanisms by which shielding occurs are reflection and refraction of sound at the boundary between the jet and the surrounding air and by diffraction around the jet.

The noise source is modelled by a stationary, discrete frequency point source located at (r_o, θ_o, z_o). The shielding jet is a cylinder of radius a, and is infinite in extent along the z-axis. The temperature and flow velocity are uniform across the cylinder cross-section. The model is illustrated in Figure 1. The expression for acoustic velocity potential is written for two regions; region I is outside the jet, region II is within the jet.

In region I (outside the flow)

$$\nabla^2\phi - \frac{1}{c_o^2}\phi_{tt} = \frac{q_o}{\rho_o} e^{-i\omega t}\,\delta(r-r_o)\,\delta(\theta-\theta_o)\,\delta(z-z_o) \qquad 1)$$

In region II (inside the flow)

$$\nabla^2\phi - M^2\phi_{zz} - \frac{2M}{c_1}\phi_{zt} - \frac{1}{c_1^2}\phi_{tt} = 0 \qquad 2)$$

Where:

$(r_o, \theta_o, z_o,)$ - location of point source

ω - source frequency

q_o - source strength

ρ - fluid density

c - sound speed

M - mach number - (jet flow speed/c_1)

$\nabla^2\phi = \phi_{rr} + \frac{1}{r}\phi_r + \frac{1}{r^2}\phi_{\theta\theta} + \phi_{zz}$

Note: The subscript o refers to conditions outside the flow (ambient), and 1 refers to conditions within the heated jet.

The boundary conditions at the interface between the ambient air and the jet are:

1) Pressure cont[illegible]ity

$$(p)_o = (p)_1 \quad \text{at } r = a$$

- or -

$$\left[-\rho_o(\phi_t)_o = -\rho_1(\phi_t + V\phi_z)_1\right] \tag{3}$$

2) Continuity of the vortex sheet (2). This condition states that the displacement of the medium is continuous and symmetrical at the boundary; r = a. Denoting this displacement by $\eta = (z, t)$, then:

$$\left.\frac{D\eta}{Dt}\right|_o = \left.\frac{D\eta}{Dt}\right|_1 \quad \text{at } r = a$$

- or -

$$(\eta_t)_o = (\eta_t + V\eta_z)_1, \quad \text{at } r = a \tag{4}$$

Eliminating time from equations 1 and 2 by assuming:

$$\phi(r, \theta, z, t) = \psi(r, \theta, z)e^{-i\omega t}$$

Then:

$$\nabla^2\psi + \frac{\omega^2}{c_o^2}\psi = \frac{q_o}{\rho_o}\delta(r-r_o)\delta(\theta-\theta_o)\delta(z-z_o) \tag{5}$$

$$\nabla^2\psi - M^2\psi_{zz} + \frac{2iM\omega}{c_1}\psi_z + \frac{\omega^2}{c_1^2}\psi = 0 \tag{6}$$

With boundary conditions

I) $$(i\omega\rho_o\psi)_o = (i\omega_o\rho_1\psi - \rho_1 V\psi_z)_1 \quad \text{at } r = a \tag{7}$$

II) with similar assumption that

$$\eta(z,t) = \eta'(z)e^{-i\omega t}$$

$$(-i\omega\eta')_o = (-i\omega\eta' + V\eta'_z)_1 \quad \text{at } r = a \tag{8}$$

Using the Fourier transform:

$$\tilde{\psi} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \psi e^{-ik_z z}\, dz$$

with corresponding inverse:

$$\psi = \int_{-\infty}^{\infty} \tilde{\psi} e^{ik_z z}\, dk_z$$

the equations become:

I) (outside flow)

$$\nabla^2\tilde{\psi} + K_o^2\tilde{\psi} = \frac{q_o e^{-ik_z z_o}}{2\pi\rho_o}\,\delta(r-r_o)\delta(\theta-\theta_o) \tag{9}$$

II) (inside flow)

$$\nabla^2\tilde{\psi} + K_1^2\tilde{\psi} = 0 \tag{10}$$

with boundary conditions

$$(i\omega\rho_o\tilde{\psi})_o = (i\rho_1\tilde{\psi}(\omega - Vk_z))_1 \quad \text{at } r = a \tag{11}$$

$$(k_o c_o\tilde{\eta})_o = (k_1 c_1\tilde{\eta})_1 \quad \text{at } r = a \tag{12}$$

where

$$k_o = \omega/c_o$$

$$k_1 = \left(\frac{\omega}{c_1} - Mk_z\right)$$

$$K_o = [k_o^2 - k_z^2]^{1/2}$$

$$K_1 = [k_1^2 - k_z^2]^{1/2}$$

The solution to the two-dimensional wave equation with a source at coordinates r_o, θ_o is given in chapter 7 of Morse & Ingard (5). Thus, the transformed acoustic potential of the wave incident on the flow cylinder is:

$$\tilde{\psi}_{in} = \frac{-iq_o e^{-ik_z z_o}}{8\pi\rho_o} \sum_{m=o}^{\infty} \epsilon_m \cos m\,(\theta-\theta_o) \begin{cases} J_m(K_o r)H_m(K_o r_o) & r < r_o \\ H_m(K_o r)J_m(K_o r_o) & r > r_o \end{cases} \qquad 13)$$

where:

$$\epsilon_m = \begin{cases} 1 & m = o \\ 2 & m \neq o \end{cases}$$

r = distance from the z-axis to the receiver

r_o = distance from the z-axis to the source

In addition to the wave emitted from the source, there is a wave scattered from the flow cylinder, of the form:

$$\tilde{\psi}_{sc} = \sum_{m=o}^{\infty} A_m H_m(K_o r) \cos m(\theta-\theta_o) \qquad 14)$$

where:

$H_m(K_o r)$ - Hankel function of the first kind = $J_M + iY_m$, which is chosen to ensure that the wave is outgoing. The wave transmitted into the flow cylinder is the solution of the homogeneous two dimensional wave equation, and is of the form:

$$\tilde{\psi}_{trans} = \sum_{m=o}^{\infty} B_m \cos m(\theta - \theta_o) J_m(K_1 r) \qquad 15)$$

The form, J_m, is chosen since the solution must be finite at the origin of the coordinate system.

Applying the boundary conditions at $r = a$, where $a < r_o$

1) Pressure continuity

$$k_o \rho_o c_o \, (\tilde{\psi}_{in} + \tilde{\psi}_{sc}) = k_1 \rho_1 c_1 \, \tilde{\psi}_{trans} \qquad \text{at } r = a \qquad 16)$$

2) Continuity of the vortex sheet

$$(\tilde{\psi}_{in})_r + (\tilde{\psi}_{sc})_r = \frac{k_o c_o}{k_1 c_1} (\tilde{\psi}_{trans})_r \qquad \text{at } r = a \qquad 17)$$

Then:

$$A_m = \frac{\alpha \varepsilon_m H_m(K_o r_o) [J_m(K_1 a) J_m'(K_o a) - T J_m(K_o a) J_m'(K_1 a)]}{[T H_m(K_o a) J_m'(K_1 a) - J_m(K_1 a) H_m'(K_o a)]}$$

where:

$$\alpha = - \frac{i q_o e^{-i k_z z_o}}{8 \pi \rho_o}$$

$$T = \frac{k_o^2 \, c_o^2 \, \rho_o \, K_1}{k_1^2 \, c_1^2 \, \rho_1 \, K_o}$$

The primes denote differentiation of the Bessel function with respect to the argument.

The transformed acoustic potential in the far field, at $r > r_o$, is:

$$\tilde{\psi} = \tilde{\psi}_{in} + \tilde{\psi}_{sc} = \alpha \sum_{m=o}^{\infty} \varepsilon_m \cos m(\theta - \theta_o) \, H_m(K_o r) \, F_m(K_o, K_1) \qquad 18)$$

where:

$$F_m(K_o,K_1) = J_m(K_or) - \frac{H_m(K_or_o)[J_m(K_1a)J_m'(K_oa) - TJ_m(K_oa)J_m'(K_1a)]}{TH_m(K_oa)J_m'(K_1a) - J_m(K_1a)H_m'(K_oa)]}$$

applying the inverse transform, and inserting the time dependence, the acoustic potential is:

$$\phi = \frac{-iq_o e^{-i\omega t}}{8\pi\rho_o} \sum_{m=o}^{\infty} \varepsilon_m \cos m(\theta-\theta_o) \int_{-\infty}^{\infty} H_m(K_or)\, F_m(K_o,K_1) e^{ik_z(z-z_o)} dk_z \qquad 19)$$

SUMMARY OF ANALYSIS TO DATE

1) Numerical Integration

A digital computer program has been written to integrate equation 19 numerically. The integration technique employed is a 36 - point Gaussian Quadrature. As a test, the technique has been used to evaluate the integral:

$$I = \int_{-\infty}^{\infty} H_o(K_or)\, e^{ik_zz}\, dk_z$$

where:

H_o = o^{th} order Hankel function

$K_o = \left\{k_o^2 - k_z^2\right\}^{1/2}$

$k_o = \omega/c_o$

This expression is the integral component of the solution for a point source located at the origin, radiating into free-space; where the receiver is located at the (cylindrical) coordinates, (r, θ, z). This expression is chosen because:

1) The integral is a constant for any combination of r and z, such that

$$\sqrt{r^2 + z^2} = \text{constant}$$

2) The magnitude of the integral decreases by one-half for each doubling of $\sqrt{r^2 + z^2}$.

The Hankel function becomes unbounded as the argument approaches zero, when $k_z = k_o$. Because of this discontinuity, the numerical integration scheme does not meet convergence criteria. Ongoing efforts in this area are concentrated in modifying the numerical integration technique to accommodate the integration near a discontinuity.

2) Simplification to Two - Dimensions

A further simplified, yet still significant model is a line source impinging on the cylinder of flow. In this model, the problem is reduced to two dimensions and the integral form of the solution is eliminated. The method of the solution is the same as for the three dimensional model. The velocity potential expressions are:

Region I (outside flow)

$$\nabla^2\phi - \frac{1}{c_o^2}\phi = \frac{a_o}{\rho_o} e^{-i\omega_1 t} \delta(r-r_o)\, \delta(\theta-\theta_o) \qquad (20)$$

Region II (inside flow)

$$\nabla^2\phi - \frac{1}{c_1^2}\phi = 0$$

where:

$$\nabla^2\phi = \phi_{rr} + \frac{1}{r}\phi_r + \frac{1}{r^2}\phi_{\theta\theta} \tag{21}$$

with boundary conditions:

1) Pressure continuity

$$-\rho_o(\phi_t)_o = -\rho_1(\phi_t)_1 \quad \text{at } r = a \tag{22}$$

2) Normal velocity continuity

$$(\phi_r)_o = (\phi_r)_1 \qquad \text{at } r = a \tag{23}$$

The wave incident upon the cylinder is

$$\phi_{in} = \frac{-iq_o e^{-i\omega t}}{\rho_o} \sum_{m=o}^{\infty} \epsilon_m \cos m\,(\theta-\theta_o) \begin{cases} J_m(k_o r)H_m(k_o r_o) & r < r_o \\ H_m(k_o r)J_m(k_o r_o) & r > r_o \end{cases}$$

The wave scattered from the cylinder is:

$$\phi_{sc} = e^{-i\omega t} \sum_{m=o}^{\infty} A_m \cos(m\theta) H_m\,(k_o r)$$

The wave transmitted into the cylinder is:

$$\phi_{trans} = e^{-i\omega t} \sum_{m=o}^{\infty} B_m \cos(m\theta)\, J_m(k_1 r)$$

where.

$$k_o = \omega/c_o$$

$$k_1 = \omega/c_1$$

The solution for the total (incident + scattered) velocity potential in the far field ($r > r_o$) is:

$$\phi = \frac{-iq_o}{\rho_o} e^{-i\omega t}\left\{\sum \epsilon_m \cos m(\theta-\theta_o)\, H_m(k_o r)\left[\overbrace{J_m(k_o r_o)}^{\text{incident}} \underbrace{- \frac{H_m(k_o r_o)\left(\frac{c_1}{c_o} J_m'(k_o a) J_m(k_1 a) - \frac{\rho_o}{\rho_1} J_m(k_o a) J_m'(k_1 a)\right)}{\left(\frac{c_1}{c_o} H_m'(k_o a)\, J_m(k_1 a) - \frac{\rho_o}{\rho_1} H_m(k_o a) J_m'(k_1 a)\right)}}_{\text{scattered}}\right]\right\} \quad 24)$$

As a check, when $\rho_1 >> \rho_o$ (flow cylinder approaches a solid cylinder) and $r_o >>$ (incident wave approaches a plane wave), the expression for the scattered wave reduces to the solution for a plane wave incident upon a cylinder as given by Morse and Ingard (Sect. 8.1 of reference (7)).

Figure 2 is a plot of the directivity index based on equation 24. The directivity is

$$DI(r_p,\beta) = 10 \log_{10} \left|\frac{P_{TOT}}{P_{REF}}\right|^2$$

where:

r_p,β - coordinates of the receiver with respect to the noise source

P_{TOT} - incident plus scattered sound pressure from equation 24.

P_{REF} - sound pressure at the location without the flow cylinder, the incident part of equation 24.

The figure is drawn for the following conditions:

a) Outside flow - ambient conditions

b) Inside flow cylinder - T = 2100° R, ρ_1 and c_1 from ideal gas relationships.

c) Jet diameter - 2a = 2.5 feet

d) r_o - Jet separation - 5.0 feet

e) r_p - radial distance from source - 40.0 feet to 1000 feet

f) Source frequency - 337. H_z.

The source frequency is evaluated based on a Strouhal Number of 0.25 for a Mach 1.5 heated jet, where the dominant source frequency is:

$$f = \frac{M \cdot c_1 \cdot St}{d}$$

where:

M - mach number =

c_1 = sound speed at 2100° R = 2250 ft.sec.

St = Strouhal number

d = jet diameter

The directivity index is plotted for distances from the noise source of 40, 125, 250, 1000. The contours show the barrier effect within the shadow zone, defined from ray acoustics as occuring between 165.5 and 194.5 degrees. The scattered wave is concentrated into side lobes distributed around the source. The lobe at 60 degrees corresponds to the critical angle from Snell's law, at which pure reflection is expected to occur.

One interesting result is that the acoustic intensity is lower directly behind the noise source (θ=o) than it is within the shadow zone. This indicates that little backscattering occurs at large angles of incidence, and that the sound wave passes through the jet.

Future work with the two - dimensional model will be directed toward comparing the results obtained by the model to experimental and other analytical results reported in the literature (7,8).

SUMMARY

A two - dimensional model of jet shielding has been developed and is currently being tested against other such models and experimental results. The purpose of development of the two - dimensional model has been to verify the process of the derivation and to provide insight into the development of the three - dimensional model.

An equation for the acoustic velocity potential in three-space has been developed. The expression is in integral form. Current efforts have been centered on development of a numerical integration scheme to solve the expression. These efforts have been unsuccessful to date, but work continues in this area. An approximate closed form solution, incorporating a form of saddle point method is also being applied to the problem. This method has been successfully applied to radiation into free space from a point source.

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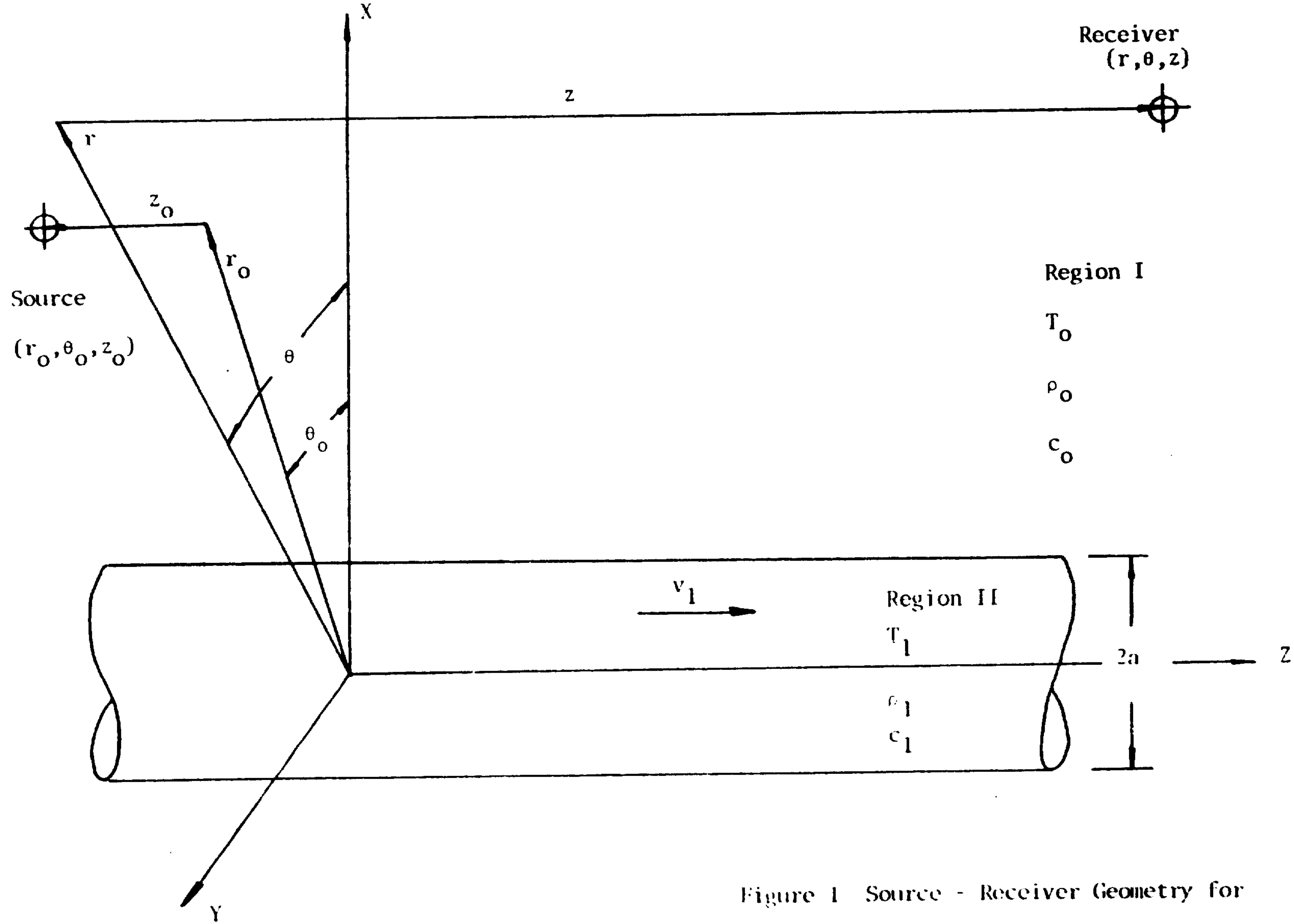

Figure 1 Source - Receiver Geometry for Jet Shielding Analysis.

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